ASYNCHRONOUS SAMPLING AND RECONSTRUCTION OF SPARSE SIGNALS

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ABSTRACT

Asynchronous signal processing is an appropriate low-power approach for the processing of bursty signals typical in biomedical applications and sensing networks. Different from the synchronous processing, based on the Shannon-Nyquist sampling theory, asynchronous processing is free of aliasing constrains and quantization error, while allowing continuous-time processing. In this paper we connect level-crossing sampling with time-encoding using asynchronous sigma delta modulators, to develop an asynchronous decomposition procedure similar to the Haar transform wavelet decomposition. Our procedure provides a way to reconstruct bounded signals, not necessarily band-limited, from related zero-crossings, and it is especially applicable to decompose sparse signals in time and to denoise them. Actual and synthetic signals are used to illustrate the advantages of the decomposer.

Index Terms— Continuous-time digital signal processing, time-encoding of signals, level-crossing sampling, asynchronous sigma delta modulators, asynchronous signal processing.

1. INTRODUCTION

The recent interest in asynchronous processing of signals is due to applications where low power consumption and continuous-time processing are essential. The range of applications of asynchronous processing goes from biomedical implants [1, 2, 3] to sensor networks in health, military and home [4], for which processing and communication is limited by power consumption. Asynchronous processing outweighs the traditional Shannon-Nyquist synchronous processing not only in power consumption but in the possible continuous-time signal processing [5, 6, 7]. Given the bursty nature of many biomedical signals, signal-dependent sampling procedures are more appropriate than uniform-sampling.

Uniform sampling resulting from the Shannon-Nyquist sampling theory cannot be implemented in many situations, for instance when the nodes of a sensor network have limited sensing and processing power, or due to sensor problems. In other situations uniform sampling is not desirable due to the required high sampling rates and complex processing. Signals collected from sensor nodes or health monitoring devices exhibit sparse nature in time in many applications. They are almost zero most of the time and changes occur on brief intervals, which challenge the analog to digital conversion given the high sampling rates required. The asynchronous sampling methods are an efficient alternative.

Uniform sampling approximates a signal by a Riemann sum, while level-crossing (LC) — a non-uniform method that reverses the roles of amplitude and time in the sampling — does the approximation by a Lebesgue sum. The significance of LC is that it follows the signal by sampling more often whenever the signal varies rapidly and less otherwise. The "opportunistic nature" of LC [8, 9, 13] is similar to the way compressive sensing deals with sparse signals [10]. Both look for compression or sparseness in the representation. Although LC sampling requires *a-priori* a set of quantization levels, typically uniform, and the samples need to be coupled with the times at which they occur, it provides a representation free of aliasing and quantization error.

A different approach, based on time-encoding, is provided by an asynchronous sigma delta modulator (ASDM), a non-linear feedback system, that represents the signal amplitude by a binary signal with zero-crossing times at different scale parameters. When comparing the LC and the ASDM sampling schemes, it can be shown that the ASDM is a LC sampler with quantization levels given by local estimates of the signal average for a certain scale. The information available in the binary signal can only provide a multi-level approximation to the signal for any particular scale setting in the ASDM. As we will show, using different scales it is possible to get representations that closely approximate the signal. Thus the idea of a decomposition procedure using ASDMs — each with different scales — is similar to the wavelet decomposition. The multilevel signals at each scale are represented by sequences of local averages and their location times providing a compressed representation of the signal. The proposed decomposition can be related to the Haar transform wavelet, which is very appropriate for multi-level signals.

Advantages of the asynchronous decomposition are: analog in nature, uses scale instead of frequency for the decomposition, and it does not suffer from aliasing — it thus applies

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to non band-limited signals. Moreover, it provides a recursive signal reconstruction of the signal from zero-crossings [11]. In this paper we illustrate the sampling, and reconstruction of sparse continuous-time signals using the proposed procedure.

2. ASYNCHRONOUS SAMPLING AND RECONSTRUCTION

The complexity of reconstructing a signal in an interval $[t_a,\ t_b]$ can be measured using the number of degrees of freedom in the sampled signal $x_s(t)$ in the interval [12]. In non-uniform sampling, it is not only necessary to have the amplitude of the samples but also their occurrence times, and as such compared with uniform sampling the reconstruction of the original signal is more complex. Level crossing (LC) sampling is a non-uniform procedure that for a given set of quantization levels it generates a non-uniform sampled signal where each sample is taken whenever the signal attains one of these quantization levels (See Fig. 1). Although reconstruction of the original signal from such sampled signal is more complex than that of a uniform sampled signal, LC sampling is less restrictive in other ways. It does not require the band-limited condition of uniform sampling and is also free of quantization. More importantly LC is signal dependent — samples are only taken when the signal is significant.

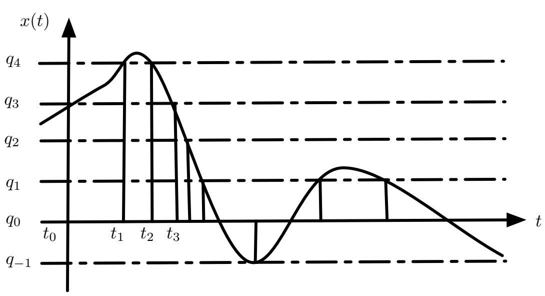

Fig. 1. Level crossing for fixed quantization levels.

A related sampling system is obtained with the asynchronous sigma delta modulator (ASDM) shown in Fig. 2, which is a nonlinear feedback system consisting of an integrator and a Schmitt trigger [2]. The ASDM maps the amplitude information of a bounded input signal $x(t)$ into a time sequence t_k, or the zero crossings of the binary output $z(t)$ of the ASDM. The bounded signal $x(t)$ and the zero-crossing times $\{t_k\}$ of the ASDM output $z(t)$ are related by the integral equation [2]

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau = (-1)^k[-b(t_{k+1}-t_k)+2\kappa\delta] \qquad (1)$$

where b, δ and κ are parameters of the ASDM. A multi-level approximation for $x(t)$, that depends on the scale parameter κ, is obtained by connecting the width of the pulses in $z(t)$

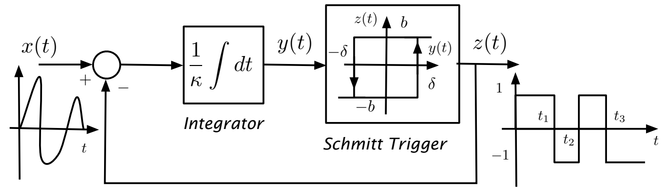

Fig. 2. Asynchronous sigma delta modulator

with local averages of the signal. This multi-level approximation can be seen as the output of a zero-order hold non-uniform sampler.

Letting $\delta = 0.5$, $b = 1$ and some κ, adding two consecutive integral equations as in (1), we have

$$\int_{t_{\kappa,k}}^{t_{\kappa,k+2}} x(\tau)d\tau = [\underbrace{(t_{\kappa,k+2}-t_{\kappa,k+1})}_{\beta_{\kappa,k}} - \underbrace{(t_{\kappa,k+1}-t_{\kappa,k})}_{\alpha_{\kappa,k}}]$$

where $\alpha_{\kappa,k}$ and $\beta_{\kappa,k}$ are defined as in Fig. 3. If we then let $T_{\kappa,k} = \beta_{\kappa,k} + \alpha_{\kappa,k}$, then the local average

$$\begin{aligned} \bar{x}_{\kappa,k} &= \frac{1}{T_{\kappa,k}}\int_{t_{\kappa,k}}^{t_{\kappa,k+1}} x(\tau)d\tau + \frac{1}{T_{\kappa,k}}\int_{t_{\kappa,k+1}}^{t_{\kappa,k+2}} x(\tau)d\tau \\ &= \frac{\alpha_{\kappa,k}-\beta_{\kappa,k}}{\alpha_{\kappa,k+\beta_{\kappa,k}}} \end{aligned} \qquad (2)$$

Thus, $\bar{x}_{\kappa,k}$ or the local average in $[t_{\kappa,k}, t_{\kappa,k+2}]$ corresponds to the difference of the areas under two consecutive pulses in $z(t)$ divided by the length of the two pulses. Using these connection between $z(t)$ and the local averages, we can obtain a multi-level approximation of $x(t)$ that would be equivalent to one using a level-crossing sampler with quantization levels $\{\bar{x}_k\}$. If we consider the $\bar{x}_k$ the best linear estimator of the signal in $[t_{\kappa,k}, t_{\kappa,k+2}]$ when no data is provided, the time-encoder can be thought of an optimal LC sampler. This would require to process the signal first with an ASDM and then to use the obtained local averages as the quantization levels for the LC.

The scale parameter κ relates to the maximum frequency of the signal. Indeed, using that $|x(t)| \leq c$ and that $b > c$, we

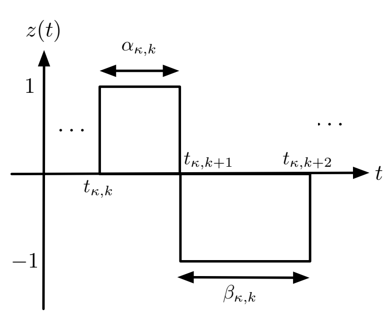

Fig. 3. The parameters $\alpha_{\kappa,k}$ and $\beta_{\kappa,k}$ in $z(t)$ for some scale κ.

obtain

$$-c(t_{k+1}-t_k) \leq \int_{t_k}^{t_{k+1}} x(\tau)d\tau \leq c(t_{k+1}-t_k)$$

Replacing (1), and solving for κ we have that

$$\frac{(b-c)(t_{k+1}-t_k)}{2\delta} < \kappa < \frac{(b+c)(t_{k+1}-t_k)}{2\delta} \tag{3}$$

In the case of non-uniform sampling, a sufficient condition for reconstruction of band-limited signals is that the maximum of $\{t_{k+1}-t_k\}$ should be less the sampling period T_s. In such a case letting $\delta = 0.5$, $b = 1$ and $b = c+\Delta$, positive $\Delta \to 0$, the relationship with the maximum frequency $f_{\max}$ of the signal is

$$\kappa \leq (2c+\Delta)T_s \leq \frac{1-0.5\Delta}{f_{\max}} \approx \frac{1}{f_{\max}}. \tag{4}$$

To obtain an expansion of the signal for different scales, just as in wavelet analysis we generate a basis for $z(t)$. The most suitable is the Haar basis which is generated from the normalized mother function

$$\psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

that generates the contracted and shifted wavelet family for integers $m \geq 0$ and $k \geq 0$,

$$\psi_{m,k}(t) = 2^{(m/2)}\ \psi(2^m t - k)$$

forming an orthonormal basis in the square–integrable space. The indice m is scaling index, k time translation and the term $2^{m/2}$ maintains a constant norm independent of scale m. This permit us to expand any signal $\zeta(t)$ in that space as partial sum that converges in the mean square metric,

$$\hat{\zeta}(t) = \sum_{m,k} \gamma_{m,k}\psi_{m,k}(t)$$

where the expansion coefficients are averages of the signal for difference scales $\{1/2^m\},\ m = 0, 1, \cdots$:

$$\gamma_{m,k} = \frac{1}{2^m}\left[\int_{t_{m,k}}^{t_{m,k+1}} \zeta(t)dt - \int_{t_{m,k+1}}^{t_{m,k+2}} \zeta(t)dt\right] \tag{6}$$

where $t_{m,k+2} - t_{m,k} = 1/2^m$ and $t_{m,k+1} - t_{m,k} = 1/2^{m+1}$. There is clearly similarity between the local averaging for some κ in equation (2) and these equations. The local averages obtained from the output of the ASDM for different scales provide an approximation to the signal. In the following section we propose a decomposition procedure similar to the Haar transform wavelet that uses the ASDM.

3. ASYNCHRONOUS DECOMPOSITION

Figure 4 displays the decomposer for three levels. At an initial scale κ_0 the output of the ASDM is used to find the corresponding local averages from which we obtain a smooth out multilevel signal using an averager and a low-pass filter. For L decomposition levels the detail signals are

$$\begin{aligned} f_{\kappa_0,1}(t) &= x(t) - d_{\kappa_0,1}(t) \\ f_{\kappa_1,2}(t) &= d_{\kappa_0,1}(t) - d_{\kappa_1,2}(t) \\ &\ \ \vdots \\ f_{\kappa_{L-1},L}(t) &= d_{\kappa_{L-2},L-1}(t) - d_{\kappa_{L-1},L}(t) \end{aligned} \tag{7}$$

with scale factors

$$\kappa_\ell = \kappa_0/2^\ell \qquad \ell = 1, \cdots, L$$

From (7) we obtain the following expansion for the signal

$$x(t) = \sum_{\ell=1}^{L} f_{\kappa_{\ell-1},\ell}(t) + d_{\kappa_{L-1},L}(t) \tag{8}$$

corresponding to the different levels with different scales.

3.1. Representation of sparse signals in time

The above decomposition is especially appropriate for sparse signals in time. Modeling a sparse signal as

$$s(t) = \sum_k \alpha_k p_\Delta(t-t_k) + \eta(t)$$
$$\text{where}\ \ p_\Delta(t) = u(t) - u(t-\Delta),\ \Delta \to 0$$

that is, $s(t)$ is a sequence of very narrow pulses located at arbitrary times t_k and embedded in noise $\eta(t)$ with a variance much smaller than that of the signal.

Assuming the noise $\eta(t)$ is zero mean, the above decomposition for an appropriate scale κ_0 will pick the narrow pulses as

$$d_1(t) = \sum_k \alpha_k p_\Delta(t-t_k)$$

and the detail signal $f_1(t) = \eta(t)$ would be the noise. For a real sparse signal, it would be necessary to consider more than one decomposition level with different scales.

4. SIMULATIONS

To illustrate the performance of the decomposer we apply it to heart sound records [14, 15] which are inherently sparse. The analyzed signal and the decomposed parts for the first three levels are shown in the left plot of Fig. 5. As we can see from the resulting spectra in the right plot of the same figure, $d(t)$ and $f(t)$ waveforms correspond to slowly and rapidly

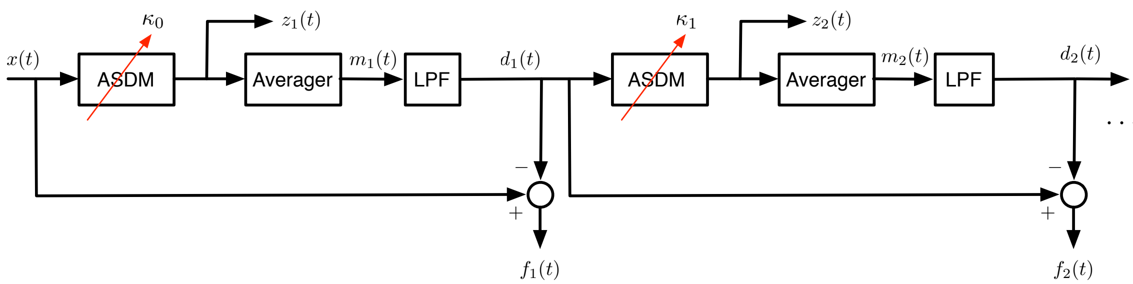

Fig. 4. Decomposer

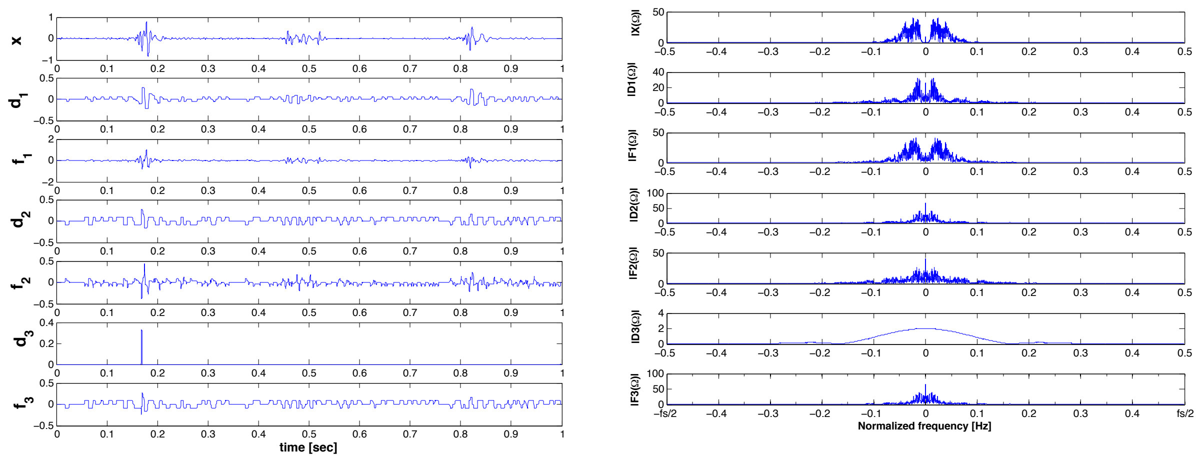

Fig. 5. Left plot: outputs of the first three levels of the decomposer to a heart sound signal (top). Right plot: Corresponding spectra.

changing parts of the signal. The component $d_1(t)$ is the local average approximation of the input signal while $f_1(t)$ is the error of this approximation. The initial value of the scale parameter, κ_0, used in the first level of the decomposer leads to averaging over a narrow window of the signal, while the smaller $\{\kappa_\ell\}$ in the subsequent levels provide averaging over a wider window for the $\{d_\ell\}$. The component $d_3(t)$ indicates the point at which the decomposition is terminated, as feeding $d_3(t)$ into another level reveals no further information. The reconstruction error, using only these three levels, is smaller than 10^{-15}.

In a second simulation, meant to stress the denoising behavior of the proposed scheme, we consider a sparse signal embedded in noise. The reconstruction performance under 30dB SNR is shown in Fig. 6. It takes only one level of decomposition with a proper κ value to accurately recover the sparse signal. This also means that the signal can be reconstructed from the resulting local averages with their width lengths. For this highly sparse signal 6.6% compression is obtained. Also approximation by using Haar wavelet resulted in 21% compression. We anticipate that this compression and signal dependent noise canceling feature is highly promising for continuous processing of bursty signals embedded in noise.

To simulate the denoising behavior for different levels of noise, 300-trial Monte Carlo simulation was performed. Noise with SNR values between -10 to 15 dB was added to the original signal and reconstructed using our algorithm. Computing the average mean-square error for each SNR we

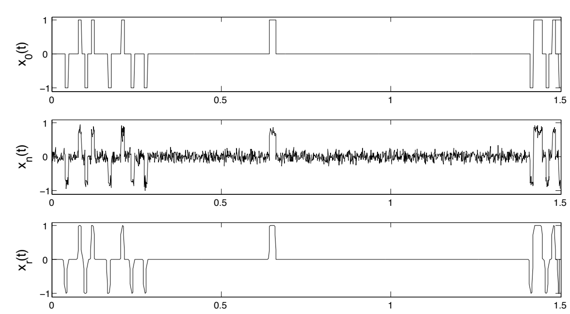

Fig. 6. Reconstruction under noise: original noise-free signal (top), noisy signal with SNR 30dB (middle), and denoised signal (bottom).

obtain the results shown in Fig. 7. The performance from SNR -10 dB and zero is significant, and for additive noise with SNR higher than 10 dB the performance of the algorithm levels off.

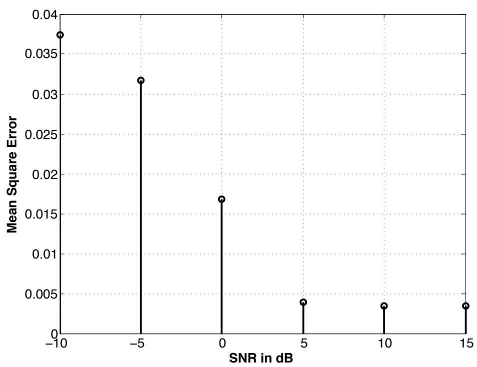

Fig. 7. Mean Square Error of the reconstructed signal for different SNR values

5. CONCLUSION

In this paper we consider asynchronous processing of sparse signals appearing in biomedical and wireless sensor applications. A scale–based representation is suggested for enabling efficient transmission of spiky or bursty data in biomedical implants or sensor networks that run on batteries. The alias-free continuous scheme exploiting the sparse behavior results in remarkable compression while asynchronous design signifies low-power dissipation. The proposed algorthm is robust to noise while having low computational complexity. The recovery results are promising given the obtained high degree of compression with adequate accuracy. Integrating our procedure with wavelets allows us to investigate the realizability of a simple yet efficient transmission and retrieval of such prevalent signals.

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